

Worksheet for 2020-09-02

Problem 1. Sketch the polar curve $r = 1 - 2 \sin \theta$. It is a limaçon with an inner loop.

- (a) Set up an integral which computes the arc length of the inner loop (not of the whole curve). The integral is sadly not doable by hand.
- (b) Find the area of the region outside the inner loop but inside of the limaçon.

Problem 2. Consider the portion of the spiral $r = \theta$ with $2\pi/3 \leq \theta \leq 5\pi/6$. See Figure 1. Compute the area *underneath* this curve in two ways:

- Convert to parametric equations and use methods of §10.2.
- First compute the area of the region with corners O, B, and D using methods of §10.4. Then use that to find the desired area. **Hint:** Think about the right triangles $\triangle BAO$ and $\triangle DCO$.

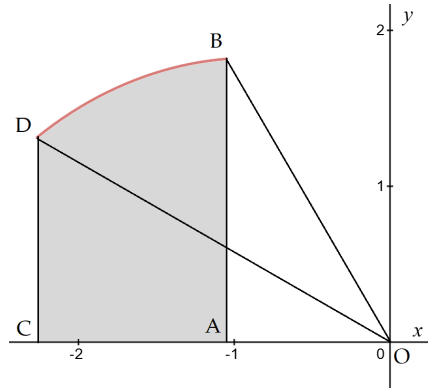


FIGURE 1. The setup of Problem 3.

Problem 3. Let $A = (0, 0, 0)$ and $B = (0, 0, 1)$, and let c be a positive real number. Consider the set of all points $P = (x, y, z)$ such that

$$|\vec{AP}| = c|\vec{PB}|.$$

Show that when $c = 1$ this set is a plane, and when $c \neq 1$ this set is a sphere.